A new method for detecting autocorrelation of evolutionary rates in large phylogenies

Qiqing Tao\textsuperscript{1,2}, Koichiro Tamura\textsuperscript{3,4}, Fabia Battistuzzi\textsuperscript{5}, and Sudhir Kumar\textsuperscript{1,2,*}

\textsuperscript{1}Institute for Genomics and Evolutionary Medicine, Temple University, Philadelphia, PA19122
\textsuperscript{2}Department of Biology, Temple University, Philadelphia, PA19122
\textsuperscript{3}Department of Biological Sciences, Tokyo Metropolitan University, Tokyo, Japan
\textsuperscript{4}Research Center for Genomics and Bioinformatics, Tokyo Metropolitan University, Tokyo, Japan
\textsuperscript{5}Department of Biological Sciences, Oakland University, Rochester, MI48309

Correspondence to:
Sudhir Kumar
Temple University
Philadelphia, PA 19122, USA
E-mail: s.kumar@temple.edu

© The Author(s) 2019. Published by Oxford University Press on behalf of the Society for Molecular Biology and Evolution. This is an Open Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (http://creativecommons.org/licenses/by-nc/4.0/), which permits non-commercial re-use, distribution, and reproduction in any medium, provided the original work is properly cited. For commercial re-use, please contact journals.permissions@oup.com
Abstract

New species arise from pre-existing species and inherit similar genomes and environments. This predicts greater similarity of the tempo of molecular evolution between direct ancestors and descendants, resulting in autocorrelation of evolutionary rates in the tree of life. Surprisingly, molecular sequence data have not confirmed this expectation, possibly because available methods lack the power to detect autocorrelated rates. Here we present a machine learning method, CorrTest, to detect the presence of rate autocorrelation in large phylogenies. CorrTest is computationally efficient and performs better than the available state-of-the-art methods. Application of CorrTest reveals extensive rate autocorrelation in DNA and amino acid sequence evolution of mammals, birds, insects, metazoans, plants, fungi, parasitic protozoans, and prokaryotes. Therefore, rate autocorrelation is a common phenomenon throughout the tree of life. These findings suggest concordance between molecular and non-molecular evolutionary patterns, and they will foster unbiased and precise dating of the tree of life.
Introduction

Rates of molecular sequence evolution vary extensively among species (Ho and Duchêne 2014; Kumar and Hedges 2016; dos Reis et al. 2016). The causes and consequences of evolutionary rate variation among species are of fundamental importance in molecular phylogenetics and systematics (Kimura 1983; Lanfear et al. 2010; Lynch 2010). They inform about the relationship among molecular, biological, and life history traits, and are a prerequisite for reliable estimation of divergence times among species and genes (Ho and Duchêne 2014; Kumar and Hedges 2016).

Three decades ago, Gillespie (1984) proposed that molecular evolutionary rates within a phylogeny will be autocorrelated due to similarities in genomes, biology and environments between ancestral species and their immediate progeny. This idea led to statistical modelling of the variability of evolutionary rates among branches and formed the basis of the earliest relaxed clock methods for estimating divergence times without assuming a strict molecular clock (Sanderson 1997; Thorne et al. 1998; Kumar 2005; Ho and Duchêne 2014; Kumar and Hedges 2016). However, the independent branch rate (IBR) model has emerged as a strong alternative to the autocorrelated branch rate (ABR) model. The IBR model posits that rates vary randomly throughout the tree, such that the evolutionary rate similarity between an ancestor and its descendant is, on average, no more than that between more distantly-related branches in a phylogeny (Drummond et al. 2006; Ho and Duchêne 2014).

The IBR model is now widely used in estimating divergence times from molecular data for diverse groups of species. It has been assumed for mammals (Drummond et al. 2006), birds (Brown et al. 2008; Claramunt and Cracraft 2015; Prum et al. 2015), amphibians (Feng et al. 2017), plants (Moore and Donoghue 2007; Bell et al. 2010; Smith et al. 2010; Linder et al. 2011; Lu et al. 2014; Barreda et al. 2015; Barba-Montoya et al. 2018), and viruses (Drummond et al. 2006; Buck et al. 2016; Metsky et al. 2017). If the IBR model best explains the variability of evolutionary rates, then we must infer a decoupling of molecular and biological evolution. This is because morphology, behavior, and other life history traits are more similar between closely-related species (Sargis and Dagosto 2008; Lanfear et al. 2010; Cox and Hautier 2015) and are correlated with taxonomic or geographic distance (Wyles et al. 1983; Shao et al. 2016).
Alternatively, the widespread use of the IBR model (Drummond et al. 2006; Moore and Donoghue 2007; Brown et al. 2008; Bell et al. 2010; Smith et al. 2010; Linder et al. 2011; Lu et al. 2014; Claramunt and Cracraft 2015; Prum et al. 2015; Buck et al. 2016; Feng et al. 2017; Metsky et al. 2017) may be due to the fact that the currently available statistical tests lack sufficient power to reject the IBR model (Ho et al. 2015). In fact, some studies report extensive branch rate autocorrelation (e.g., Lepage et al. 2007), but others do not agree (e.g., Linder et al. 2011).

Consequently, many researchers use both ABR and IBR models when applying Bayesian methods to date divergences (Wikström et al. 2001; Drummond et al. 2006; Bell et al. 2010; Erwin et al. 2011; Meredith et al. 2011; dos Reis et al. 2012; Magallón et al. 2013; Jarvis et al. 2014; Hertweck et al. 2015; dos Reis et al. 2015; Foster et al. 2016; Liu et al. 2017; Pacheco et al. 2018; dos Reis et al. 2018; Takezaki 2018). This practice can result in widely differing time estimates under ABR and IBR models, which makes biological interpretation challenging (Battistuzzi et al. 2010; Christin et al. 2014; dos Reis et al. 2014; dos Reis et al. 2015; Foster et al. 2016; Liu et al. 2017; Pacheco et al. 2018; Takezaki 2018). For example, as compared to the ABR model, the use of IBR model has been reported to produce 66% older estimates of divergence times for two major groups of grasses (Christin et al. 2014), 30% older divergence estimate for the origin of a major group of mammal (Erinaceidea) (Meredith et al. 2011), and 50% younger estimates for two clades of parasitic protozoans in birds (Pacheco et al. 2018). The choice of branch rate model also strongly influences posterior credibility intervals, because these intervals are often wider under the ABR model (Battistuzzi et al. 2010).

Therefore, we need a powerful method to accurately test whether evolutionary rates are autocorrelated in a phylogeny. Application of this method to molecular datasets representing taxonomic diversity across the tree of life will enable an assessment of the preponderance of autocorrelated rates in nature. Here, we introduce a new machine learning approach (CorrTest) that shows high power to detect autocorrelation between molecular branch rates. CorrTest is computationally efficient, and its application to a large number of datasets establishes the pervasiveness of rate autocorrelation in the tree of life.
**New Method**

Machine learning (McL) is widely used to solve problems in many fields, including ecology (Christin et al. 2018; Willcock et al. 2018) and population genetics (Saminadin-Peter et al. 2012; Schrider and Kern 2016; Schrider and Kern 2018). We present a supervised McL framework (Bzdok et al. 2018) used to build a predictive model that distinguishes between ABR and IBR models, a major challenge in molecular phylogenetics and phylogenomics. In our McL approach, the input is a molecular phylogeny with branch lengths and the output is a classification that corresponds to whether or not the evolutionary rates in the phylogeny are autocorrelated among branches (ABR or IBR, respectively). An overview of our McL approach is presented in figure 1.

To build a predictive model, McL needs measurable properties (features) that can be derived from the input data (phylogeny with branch lengths). The selection of informative and discriminating features (Fig. 1g and h) is critical to the success of McL. We derive relative lineage rates using a given molecular phylogeny with branch (“edge”) lengths (Fig. 1e and f) by using Tamura et al.’s (2018) method, and use these relative rates to generate informative features. The use of the relative rate framework (RRF) is necessary because we cannot derive branch rates without knowing node times in the phylogeny. For example, we need to know node times t’s in figure 2 to convert branch lengths into branch rates, but these node times are what investigators wish to estimate by using a Bayesian approach that requires selection of a branch rate model. In contrast, the estimation of relative lineage rates does not require knowledge of divergence times. This is because an evolutionary lineage includes all the branches in the descendant subtree (e.g., lineage a contains branches with lengths $b_1$, $b_2$, and $b_5$ in Fig. 2) and the relative rate between sister lineages is simply the ratio of the evolutionary depths of the two lineages (Tamura et al. 2018). In figure 2, $R_a$ and $R_b$ are two lineage rates whose relative value can be estimated by the ratio of lineage lengths $L_a$ and $L_b$, where the lineage length is a function of lengths of all branches in the subtree. Tamura et al. (2018) presented RRF to estimate these relative lineage rates analytically by using branch lengths only. Furthermore, Tamura et al.’s (2018) method generates relative lineage rates such that all the lineage rates in a phylogeny are relative to the rate of the ingroup root.
lineage (R₀, Fig. 2). Use of RRF enabled us to develop a number of features for building a McL predictive model.

We considered the correlation between ancestral and descendant lineage rates (ρ_ad), the correlation between the sister lineage rates (ρ_s), and the decay in ρ_ad when one or two intervening branches are skipped (d₁ and d₂, respectively) as features (see Materials and Methods). ρ_ad was considered as a feature because our analyses of simulated data showed that ρ_ad was much higher for phylogenetic trees in which molecular sequences evolved under an ABR model (0.96) than an IBR model (0.54, Fig. 3a). Importantly, ρ_ad is not expected to be zero under the IBR model because ρ_ad is a correlation between ancestral and descendant lineages, not independent branches. An ancestral lineage includes all the lineages in the descendant subtree, therefore, the evolutionary rate of an ancestral lineage naturally depends on the evolutionary rates of its descendant lineages in RRF (Tamura et al. 2018). Therefore, ancestral and descendant lineage rates will be correlated. While ρ_ad is greater than zero, it showed distinct patterns for ABR and IBR models and is, thus, a good candidate feature for McL (Fig. 3a).

As our second feature, we considered the correlation between the sister lineages (ρ_s), because ρ_s was higher for the ABR model (0.89) than the IBR model (0.00, Fig. 3b). Two additional features considered were the decay in ρ_ad when one or two intervening branches are skipped (d₁ and d₂, respectively). We expect that ρ_ad will decay more slowly under ABR than IBR, which was confirmed (Fig. 3c). The selected set of candidate features (ρ_s, ρ_ad, d₁, and d₂) can be measured for any phylogeny with branch lengths, e.g., derived from molecular data using the maximum likelihood method. They are then used to train the McL classifier (Fig. 1i and j). For this purpose, we need a large set of phylogenies in which branch rates are autocorrelated for which the numerical state 1 is assigned as true positive case (Fig. 1d) and phylogenies in which the branch rates are independent for which the numerical state 0 is assigned as true negative case (Fig. 1c).

However, there is a paucity of empirical data for which ABR and IBR are firmly established. We, therefore, trained our McL model on a simulated dataset, a common practice in McL applications when reliable real world training datasets are few in number (Saminadin-Peter et al. 2012; Schrider and Kern 2016; Ekbatani et al. 2017; Le et al.
We used computer simulations to generate 1,000 molecular datasets that evolved with ABR models and 1,000 molecular datasets that evolved with IBR models (Fig. 1a and b). To ensure the general utility of our model for analyses of diverse data, we simulated molecular sequences with varying numbers of species, degrees of rate autocorrelation, diversity of evolutionary rates and substitution pattern parameters (see Materials and Methods). Candidate features (ρa, ρad, d1, and d2) were computed for all 2,000 training datasets (Fig. 1g and h), each of which was associated with a numerical output state (0 and 1 for IBR and ABR, respectively; Fig. 1c and d). These features were used to build a predictive model by employing a logistic regression (Fig. 1j). This predictive model was then used to generate a correlation score (CorrScore) for any phylogeny with branch lengths.

We also developed a conventional statistical test (CorrTest), based on CorrScore (0 - 1), to provide a P-value to decide whether the IBR model should be rejected. A high CorrScore indicates a high probability that the branch rates are autocorrelated. At a CorrScore greater than 0.5, Type I error (rejecting IBR when it was true) was less than 5%. Type I error of 1% (P-value of 0.01) was achieved with a CorrScore greater than 0.83 (Fig. 3e). CorrTest is available at Github (https://github.com/cathyqqtao/CorrTest) and in the MEGA X software (Kumar et al. 2018).

RESULTS

We evaluated the sensitivity and specificity of our predictive model using receiver operating characteristic (ROC) curves. They measured the sensitivity of our method to detect rate autocorrelation when it was present (true positive rate, TPR) and when it was not present (false positive rate, FPR) at different CorrScore thresholds. TPR = TP/(TP + FN) and FRP = FP/(TN + FP), where TP, FN, FP, and TN stand for true positives, false negatives, false positives, and true negatives, respectively. The ROC curve for McL using all four features was the best, which led to the inclusion of all four features in the predictive model (Fig. 3d; Material and Methods). The area under the ROC (AUROC) was 99%, with a 95% TPR (i.e., ABR detection) achieved at the expense of only 5% FPR (Fig. 3d, black line). The area under the precision recall (AUPR) curve was also extremely high (0.99; Fig. 3d inset), where precision and recall were defined as TP/(TP+FP) and TP/(TP + FN).
(=TPR), respectively. It suggested that CorrTest detects the presence of rate autocorrelation with very high accuracy (= (TP+TN)/(TP + FP + FN + TN)) and precision.

We also performed standard cross-validation tests (Fig. 1k) using the simulated data to evaluate the accuracy of the predictive models when only a subset of data are used for training. In the 10-fold cross-validation, the predictive model was developed using 90% of the simulated training datasets, and then its performance was tested on the remaining 10% of the datasets. The AUROC was greater than 0.99 and the accuracy was high (>94%). Even in the 2-fold cross-validation, where only half of the datasets (500 ABR and 500 IBR datasets) were used for training the model, leaving the remaining half for testing, the AUROC was greater than 0.99 and the classification accuracy was greater than 92%. This suggested that the predictive model is robust to the size of the training set used.

We tested the performance of CorrTest on a large collection of simulated datasets where the correct rate model is known (Fig. 1l). In these datasets (Tamura et al. 2012), different software and simulation schemes were used to generate sequences with a wide range of empirically derived G+C contents, transversion/transition ratios, and evolutionary rates under both ABR and IBR models (see Materials and Methods). CorrTest accuracy was greater than 94% in detecting ABR and IBR correctly for datasets that were simulated with low and high G+C contents (Fig. 4a), small and large transition/transversion ratios (Fig. 4b), and different rates of evolution (Fig. 4c). As expected, CorrTest performed best on datasets that contained more and longer sequences (Fig. 4d).

In the above analyses, we used the correct tree topology and nucleotide substitution model (Hasegawa-Kishino-Yano (HKY) model (Hasegawa et al. 1985) with 5 discrete gamma categories). We relaxed this requirement and evaluated CorrTest by inferring the tree topology and branch lengths using the neighbor-joining method (Saitou and Nei 1987) with an oversimplified Kimura's (1980) two-parameter substitution model. Therefore, the estimation of the total number of substitutions between sequences were biased because inequality of nucleotide frequencies and variation of evolutionary rate across sites were not considered. Naturally, many inferred phylogenies contained topological errors, but we found the accuracy of CorrTest to be high as long as the dataset contained >100 sequences of length >1,000 base pairs (Fig. 4e). CorrTest also performed
well even when 20% of the non-trivial tree bi-partitions were incorrect in the inferred phylogeny (Fig. 4f, see Materials and Methods). Therefore, CorrTest will be most reliable for large datasets, and is relatively robust to errors in phylogenetic inference.

**CorrTest versus Bayes factor analysis**

We compared the performance of CorrTest with that of the Bayes factor approach. Because the Bayes factor method is computationally demanding, we limited our comparison to 100 datasets containing 100 sequences each (see Material and Methods). We computed Bayes factors (BF) by using the stepping-stone sampling (SS) method (see Materials and Methods). BF-SS analysis detected autocorrelation ($P < 0.05$) for 33% of the ABR datasets (Fig. 5a, red curve in the ABR zone). Marginal log-likelihoods under the ABR model were very similar to or lower than those for the IBR model, which led to the failure to detect autocorrelation for 67% of ABR datasets. Therefore, BF-SS was conservative in rejecting the IBR model, as has been reported (Ho et al. 2015). CorrTest correctly detected the ABR model for 88% of the datasets ($P < 0.05$; Fig. 5b, red curve in ABR zone). For IBR datasets, BF-SS correctly detected the IBR model for 89% (Fig. 5a, blue curve in the IBR zone), whereas CorrTest correctly detected IBR model for 86% (Fig. 5b, blue curve in the IBR zone). Therefore, BF-SS performs well in correctly classifying phylogenies that evolve under an IBR model, but not an ABR model. The power of CorrTest to correctly inferring the ABR model is responsible for its higher overall accuracy (87% vs. 61% for BF-SS). Such a difference in accuracy was observed at different levels of statistical significance (Fig. 5c) for datasets that evolved with high ($v < 0.1$), moderate ($0.1 \leq v < 0.2$) and low ($v \geq 0.2$) degree of rate autocorrelation (Fig. 5d), where $v$ is the parameter controlling the degree of rate autocorrelation (Kishino et al. 2001). However, the accuracy of CorrTest and BF-SS was similar in detecting IBR (Fig. 5e). The accuracy was slightly higher for CorrTest than BF-SS for phylogenies with high (standard deviation $\geq 0.3$) and low (standard deviation $< 0.2$) degree of independent rate variation, but the reverse was true for phylogenies with moderate ($0.2 \leq$ standard deviation $< 0.3$) degree of independent rate variation. These comparisons suggest that the McL method enables highly accurate detection of rate autocorrelation in a given phylogeny and presents an alternative to Bayes factor analyses for large datasets.
Autocorrelation of rates is common in molecular evolution

The high accuracy and fast computational speed of CorrTest enabled us to test the presence of autocorrelation in 17 large datasets from 11 published studies of eukaryotic species and 2 published studies of prokaryotic species encompassing diverse groups across the tree life. This included nuclear, mitochondrial and plastid DNA, and protein sequences from mammals, birds, insects, metazoans, plants, fungi, parasitic protozoans and prokaryotes (Table 1). CorrTest rejected the IBR model for all datasets ($P < 0.05$). In these analyses, a time-reversible process was assumed for substitutions of nucleotides and amino acids in the original studies (Table 1). However, the violation of this assumption may produce biased results in phylogenetic analysis (Jayaswal et al. 2014). We therefore applied an unrestricted substitution model (Yang 1994) for analyzing all the nucleotide datasets and found that CorrTest rejected the IBR model in every case ($P < 0.05$). This robustness stems from the fact that the branch lengths estimated under the time-reversible and the unrestricted model are highly correlated for these data ($r^2 > 0.99$). This could be the reason why CorrTest produced reliable results even when an oversimplified model (Kimura 1980) was used for analyzing computer simulated data (Fig. 4e and f).

These results suggest that the autocorrelation of rates among lineages is very common in molecular phylogenies. This pattern contrasts starkly with those reported in many previous studies (Drummond et al. 2006; Moore and Donoghue 2007; Brown et al. 2008; Bell et al. 2010; Smith et al. 2010; Linder et al. 2011; Jarvis et al. 2014; Lu et al. 2014; Barreda et al. 2015; Claramunt and Cracraft 2015; Prum et al. 2015; Feng et al. 2017; Barba-Montoya et al. 2018). In fact, all but three datasets (Battistuzzi and Hedges 2009; Erwin et al. 2011; Calteau et al. 2014) received very high CorrScores, resulting in extremely significant $P$-values ($P < 0.01$). The IBR model was also rejected for the three datasets ($P < 0.05$), but their CorrScores were not as high, likely because of limited or biased sampling of the evolutionary diversity. For example, the metazoan dataset (Erwin et al. 2011) contains sequences primarily from highly divergent species that share common ancestors hundreds of millions of years ago. In this case, tip branches in the phylogeny are long and their evolutionary rates are influenced by many un-sampled lineages. Such sampling effects weaken the rate autocorrelation signal. We verified this behavior via an analysis of simulated data and found that CorrScores decreased when
density of taxon sampling was lower (Fig. 6). Overall, CorrTest detected rate autocorrelation in all the empirical datasets.

**Magnitude of rate autocorrelation in molecular data**

CorrScore is influenced by the size of the dataset in addition to the degree of autocorrelation, so it is not a direct measure of the degree of rate autocorrelation (effect size) in a phylogeny. Instead, one should use a Bayesian approach to estimate the degree of rate autocorrelation, for example, under Kishino et al.’s (2001) autocorrelated rate model. In this model, a single parameter (ν) captures the degree of autocorrelation among branches in a phylogenetic tree. A low value of ν indicates high autocorrelation, so, we use the inverse of ν to represent the degree of rate autocorrelation. MCMCTree (Yang 2007) analyses of 100 simulated datasets (see Materials and Methods) confirmed that the estimated ν is related linearly with the true value (Fig. 7). Based on the results from the analysis of empirical datasets, we suggest that 1/ν greater than 3 be considered high autocorrelation, 1/ν between 1 and 3 be considered moderate autocorrelation, and 1/ν below 1 be considered weak autocorrelation. Based on this ad hoc criterion, we may conclude that rate autocorrelation is moderate to high for empirical datasets examined for species across the tree of life.

Other interesting patterns emerge from this analysis. First, rate autocorrelation is highly significant for mutational rates (= substitution rate at neutral positions), which are expected to be similar in sister species because they inherit cellular machinery from a common ancestor (Table 1). The substitution rates at the third codon positions and the four-fold degenerate sites are considered to be a good proxy of synonymous substitution rate, because they are largely neutral and are the best reflection of mutation rates (Kumar and Subramanian 2002). For example, the mammalian datasets A and B, which consisted of the four-fold degenerate sites and the third codon positions, received high CorrScores of 0.99 and 0.98, respectively (P < 0.001). Second, our model detected a strong signal of autocorrelation among amino acid substitution rates, which are dictated by natural selection (Table 1). For example, mammalian dataset C received a high CorrScore of 0.99 in the proteins encoded in the same genes in the datasets of third codon positions (mammalian dataset B) and four-fold degenerate sites (mammalian dataset A). Bayesian analyses also showed that the degree of rate autocorrelation is similar: inverse of ν was
3.21 in four-fold degenerate sites and 3.11 in amino acid sequences for mammalian datasets. Third, mutational and substitution rates in nuclear genomes and substitution rates in mitochondrial genomes are highly autocorrelated ($P < 0.05$, Table 1) (synonymous substitution rate was not used for mitochondrial data). These results establish that molecular and non-molecular evolutionary patterns are concordant, because morphological characteristics are correlated with taxonomic or geographic distance (Wyles et al. 1983; Sargis and Dagosto 2008; Lanfear et al. 2010; Cox and Hautier 2015; Shao et al. 2016).

**DISCUSSION**

Our results demonstrate that a McL framework is useful to develop a method to detect the presence of rate autocorrelation among branches in a phylogeny. This method yields CorrScore estimates that enabled development of a conventional statistical test (CorrTest) to detect autocorrelation. This method can be used for datasets with small (50 - 100) and large numbers of sequences, as supported by high accuracy achieved by CorrTest in the analysis of simulated datasets (Fig. 4). We also evaluated if higher accuracy could be achieved by building specific predictive models that were trained separately using data with different ranges of the number of sequences (n): M100 (n ≤ 100), M200 (100 < n ≤ 200), M300 (200 < n ≤ 300), and M400 (n > 300). A specific threshold for CorrScore that corresponded to certain $P$-value was determined for each training subset and then tested using Tamura et al.’s (2012) simulated data with the corresponding number of sequences. For example, we used the threshold determined for the model trained with small data (M100) on the test data that contain less than 100 sequences, and used the threshold determined for the model trained with large data (M400) on the large test data (400 sequences). We found that the accuracy obtained by using the specific thresholds determined for datasets with different numbers of sequences (M100 - M400) (Fig. 8) was similar to the accuracy obtained by using a global threshold (Fig. 4d - f). This is because the McL algorithm automatically incorporated the impact of the number of sequences when determining the relationship of four selected features ($\rho_{ad}$, $\rho_{s}$, $d_1$ and $d_2$). This justifies the usage of the globally trained CorrTest that we used in all the empirical analyses reported here.
No single branch rate model may be adequate for Bayesian dating analyses, and one may need to use a mixture of models because different groups of species and genes in a large phylogeny may have evolved with different levels of autocorrelation (e.g., Lartillot et al. 2016 and Tamura et al. 2018). In this sense, results produced by CorrTest (and by Bayes factor) analyses primarily detect the presence of rate autocorrelation, but they do not tell us if the rate autocorrelation exists in every clade of a phylogeny or if the degree of autocorrelation is the same in all the clades. One may apply CorrTest to individual clades (subtrees) to evaluate these patterns. For example, we divided a few large empirical phylogenies (Meredith et al. 2011; dos Reis et al. 2012; Misof et al. 2014; Prum et al. 2015) into subtrees with at least 50 sequences, and applied CorrTest on subtrees to detect the existence of clade-specific rate autocorrelation. These analyses showed a wide range of $1/\nu$ values, which was consistent with the large range of the autocorrelation parameter values observed for different datasets we analyzed ($1.2 < 1/\nu < 40$, Table 1). That is, the degree of autocorrelation likely varies among different types of genes, different types of substitutions, and in different taxonomic groups. In the future, it will be useful to identify such patterns at micro- and macro-evolutionary scales and to elucidate mechanistic underpinnings of the differences observed.

**CONCLUSION**

We have presented a fast, scalable, and accurate method (CorrTest) to detect the presence of branch rate autocorrelation in a phylogeny. In addition to molecular data, CorrTest may be used for testing autocorrelation of rates in non-molecular data, e.g., morphological characteristics, because the features required for CorrTest can be calculated for any phylogeny with branch lengths. The application of CorrTest to a large number of datasets addressed an enduring question in evolutionary biology: are the molecular rates of change between species correlated or independent? We find that the rate autocorrelation is the rule, rather than the exception. So, it will be best to employ an autocorrelated branch rate model in molecular dating analyses in studies of biodiversity, phylogeography, development, and genome evolution. However, when in doubt, one may conduct CorrTest, which is particularly effective for analyzing large datasets. We also expect CorrTest to be useful in analyzing many other large datasets, revealing both the extent of autocorrelated evolutionary rates in the tree of life and the exceptions to this
rule. Discovery of genes, gene families, and species groups in which branch rates are evolving without significant autocorrelation will be precursors to elucidating mechanistic underpinnings of new biological phenomena.

**Materials and Methods**

*Machine learning (McL) model*

**Training data for McL.** We simulated nucleotide alignments using independent branch rate (IBR) and autocorrelated branch rate (ABR) models using the NELSI package (Ho et al. 2015) with a variety of empirically-derived parameter values and parameters used in previous studies (Rosenberg and Kumar 2003; Ho et al. 2015). In IBR cases, branch-specific rates were drawn from a lognormal distribution with a mean gene-by-gene substitution rate and a standard deviation (in log-scale) that varied from 0.1 to 0.4, previously used in a study simulating independent rates with different levels of variation (Ho et al. 2015). In ABR cases, branch-specific rates were simulated under an autocorrelated process (Kishino et al. 2001), using equation 10.9 in Yang (2014). The initial rate was set as the mean rate derived from an empirical gene and an autocorrelated parameter, $\nu$, that was randomly chosen from a uniform distribution ranging from 0.01 to 0.3, following a previous simulation of low, moderate and high degree of rate autocorrelation (Ho et al. 2015). We used SeqGen (Grassly et al. 1997) to generate alignments under the Hasegawa-Kishino-Yano (HKY) model (Hasegawa et al. 1985) with 4 discrete gamma categories. This process used a master phylogeny, consisting of 60-400 ingroup taxa randomly sampled from the bony-vertebrate clade in the Timetree of Life (Hedges and Kumar 2009). Mean evolutionary rates, G+C contents, transition/transversion ratios and numbers of sites for simulation were derived from empirical distributions (Rosenberg and Kumar 2003). 1,000 molecular datasets were generated under ABR and IBR models separately and these 2,000 simulated datasets were used as training data in building the McL model.

**Calculation of features for McL.** Lineage-specific rate estimates ($R$'s) were obtained using equations [28] - [31] and [34] - [39] in Tamura et al. (2018). For any given node in the phylogeny (e.g., node 5 in Fig. 2), we extracted the relative rates of its ancestral lineage (e.g., $R_a$ in Fig. 2) and two direct descendant lineages (e.g., $R_1$ and $R_2$ in Fig. 2). Then,
we calculated correlation between the ancestral lineage and its direct descendant lineage rate to obtain estimates of ancestor-descendant rate correlation (ρ_{ad}). We also calculated correlation between sister lineage rates (ρ_s). We need to assign labels to lineage rates of each sister pair to determine which lineage is the 1\textsuperscript{st} sister lineage and which lineage is the 2\textsuperscript{nd} sister lineage, e.g., (R_1 and R_2) or (R_2 and R_1) in Fig. 2. If rates of the 1\textsuperscript{st} sister lineages are always higher than rates of the 2\textsuperscript{nd} sister lineages, an artificial correlation will be generated between sister lineage rates. To avoid this possibility, we randomly labeled sister lineages. The labeling of sister pairs have negligible impact (<2\%) on ρ_s when the number of sequences in the phylogeny is not too small (>50). For smaller datasets, we found that it is best to generate multiple ρ_s estimates, each using randomly labelled sister pairs, to eliminate bias that may result from the arbitrary designation of sister pairs. In this case, we recommend using the mean ρ_s from multiple replicates in the CorrTest analysis.

To avoid the assumption of linear correlation between lineages, we used Spearman rank correlation because it can detect both linear and non-linear correlation between two vectors. Two additional features were included in McL model: d_1 and d_2, which are the decay of ρ_{ad} when one or two intervening branches are skipped. We first estimated ρ_{ad\_skip1} as the correlation between rates where the ancestor and descendant were separated by one intervening branch, and ρ_{ad\_skip2} as the correlation between rates where the ancestor and descendant were separated by two intervening branches. This skipping reduces ancestor-descendant correlation, which we then used to derive the decay of correlation values by using equations d_1 = (ρ_{ad} - ρ_{ad\_skip1})/ρ_{ad} and d_2 = (ρ_{ad} - ρ_{ad\_skip2})/ρ_{ad}. These two features improved the accuracy of our model slightly. In the analyses of empirical datasets, we found that a large amount of missing data (>50\%) can result in unreliable estimates of branch lengths and other phylogenetic errors (Wiens and Moen 2008; Lemmon et al. 2009; Filipski et al. 2014; Xi et al. 2015; Marin and Hedges 2018). In this case, we recommend computing selected features (ρ_s, ρ_{ad}, d_1 and d_2) using only those lineage pairs for which >50\% of the positions contain valid data, or removing sequences with a large amount of missing data.

**Building the McL predictive model.** We trained a predictive model with only ρ_{ad}, only ρ_s or all four features (ρ_s, ρ_{ad}, d_1 and d_2) using 2,000 simulated training datasets (1,000 with ABR model and 1,000 with IBR model). For each set of training data, we inferred the
branch lengths from the molecular sequences with a fixed topology first and used these inferred branch lengths to estimate relative lineage rates for computing selected features. A numerical state of 1 was given to true positive cases (autocorrelated rates) and 0 was assigned to true negative cases (independent rates). Then, a predictive model was generated via logistic regression in the skit-learn model (Pedregosa et al. 2011), which is a python toolbox for data mining and data analysis using machine learning algorithms. This model contains the relationship between the numerical state and the selected features. Therefore, for any phylogeny with branch lengths, we can calculate features and apply the predictive model to generate a numerical output value between 0 and 1. The resulting value is referred as the correlation score (CorrScore). A high CorrScore suggests that the rates are more likely to be autocorrelated. Every CorrScore associates with a Type I error (P-value), which is the percentage of IBR cases that are incorrectly predicted as ABR. We found that Type I error of 5% (P-value of 0.05) was achieved with a CorrScore greater than 0.5, and Type I error of 1% was achieved with a CorrScore greater than 0.83. Therefore, we developed a conventional statistical test (CorrTest) based on CorrScore. CorrScores of 0.5 and 0.83 were used as the global thresholds at 5% and 1% significant levels. Using the same procedure, we also trained specific predictive models using training data with different numbers of sequences (n): M100 (n ≤ 100), M200 (100 < n ≤ 200), M300 (200 < n ≤ 300), and M400 (n > 300) and determined specific threshold for CorrScore for each model. CorrScores of 0.69, 0.61, 0.57 and 0.31 were thresholds for M100, M200, M300 and M400 at 5% significant level, respectively. CorrScores of 0.84, 0.86, 0.88 and 0.73 were thresholds for M100, M200, M300 and M400 at 1% significant level, respectively.

Test datasets
Tamura et al.’s (2012) simulated datasets were used to evaluate CorrTest’s performance. This allowed us to test the performance of our method on ABR and IBR datasets with different G + C contents (range 39 - 82%), transition/transversion ratios (range 1.9 - 6.0), and evolutionary rates (range 1.35 - 2.60 substitution per site per billion years). In IBR simulations, Tamura et al. (2012) used a uniform distribution in which branch rates were sampled from a uniform density in the interval \[((1-x).r - (1+x).r, \) where \( r \) is the mean evolutionary rate and the \( x \) is the degree of rate variation (0.5 or 1.0 for 50% and 100%
rate variation). For ABR simulations, Tamura et al. (2012) used Kishino et al.’s (2001) model with $\nu = 1$. In both scenarios, sequences were simulated on a master phylogeny of 400 ingroup taxa using the HKY substitution model with 5 discrete gamma categories. We analyzed 100 datasets simulated using the ABR model and 100 datasets simulated using the IBR model (50% rate variation). We also randomly sampled 50, 100, 200, and 300 sequences from the full set of 400 ingroup sequences, and conducted CorrTest using the correct topology and error-prone topology inferred by the neighbor-joining method (Saitou and Nei 1987) with an oversimplified substitution model of Kimura (1980) with both global and specific CorrScore thresholds. The percentage of incorrect inferred tree bi-partitions (clades) was calculated by $d/(2(m-3))$ where $d$ was the Robinson and Foulds’s (1981) topological distance between inferred and true topologies and $m$ was the number of sequences. In addition, we also tested CorrTest’s performance on 100 datasets simulated by Tamura et al. (2012) under an IBR model with 100% rate variation. CorrTest worked perfectly (100% accuracy) for these datasets (results not shown).

In addition to above analyses, we conducted another set of simulations to generate 100 datasets using IBR (independent lognormal distribution) and ABR (autocorrelated lognormal distribution) (Kishino et al. 2001) models, each using the same strategy as in training data simulation (described above) on a master phylogeny of 100 taxa randomly sampled from the bony-vertebrate clade in the Timetree of Life (Hedges and Kumar 2009). These 200 datasets were used to conduct CorrTest and Bayes factor analyses and to obtain the autocorrelation parameter ($\nu$) in MCMCTree (Yang 2007).

**CorrTest analyses**

All CorrTest analyses were conducted using customized R code (available at https://github.com/cathyqtao/CorrTest). We first estimated branch lengths of a phylogeny for sequence alignments using the maximum likelihood method with the correct substitution model and the correct topology in MEGA 7 command line version (Kumar et al. 2012; Kumar et al. 2016). We used neighbor-joining method to estimate topology and branch lengths with Kimura’s (1980) two-parameter substitution model and without the assumption of rate variation across sites under the gamma distribution in MEGA 7 command line version, when we tested the robustness of our model to topological error. We then used the estimated branch lengths to compute relative lineage
rates using RRF (Tamura et al. 2012; Tamura et al. 2018) and calculated the value of selected features ($\rho_s$, $\rho_{ad}$, $d_1$ and $d_2$) to obtain the CorrScore. We conducted CorrTest on the CorrScore to estimate the $P$-value of detecting rate autocorrelation. No calibration was needed for CorrTest analyses. CorrTest is also available in the MEGA X software (Kumar et al. 2018).

**Bayes factor analyses**

We computed the Bayes factor via stepping-stone sampling (BF-SS) (Xie et al. 2011) with $n = 20$ and $a = 5$ using mcmc3r (dos Reis et al. 2018). BF-SS estimates the marginal likelihoods using the idea from importance sampling, a common practice in statistics, to construct a path between prior and posterior distributions of a model (Xie et al. 2011; Baele et al. 2013). We chose BF-SS because the harmonic mean estimator has many statistical shortcomings (Lepage et al. 2007; Xie et al. 2011; Baele et al. 2013) and thermodynamic integration (Lartillot and Philippe 2006) is less efficient than BF-SS (Baele et al. 2012). For each dataset, we computed the log-likelihoods ($lnK$) under the IBR and ABR models. The Bayes factor posterior probability for ABR was calculated as shown in dos Reis et al. (2018). We used only one calibration point at the root (true age with a narrow uniform distribution) in all the Bayesian analyses, as it is the minimum number of calibrations required by MCMCTree (Yang 2007). For other priors, we used diffused distributions of “rgene$\_gamma = 1$ 1”, “sigma2$\_gamma=1$ 1” and “BDparas = 1 1 0”. In all Bayesian analyses, two independent runs of 5,000,000 generations each were conducted, and results were checked in Tracer (Rambaut et al. 2014) for convergence. ESS values were higher than 200 after removing 10% burn-in samples for each run.

**Analysis of empirical datasets**

We used 17 datasets from 11 published studies of eukaryotes and 2 published studies of prokaryotes that cover the major groups in the tree of life (Table 1). These data were selected for relative completeness (missing data <50%) and sample size (>80 sequences). As we know, a large amount of missing data (>50%) can result in unreliable estimates of branch lengths and other phylogenetic errors (Wiens and Moen 2008; Lemmon et al. 2009; Filipski et al. 2014; Xi et al. 2015; Marin and Hedges 2018) and potentially bias CorrTest results. When a phylogeny with branch lengths was available...
from the original study, we estimated relative rates directly from the branch lengths via RRF (Tamura et al. 2018) and computed selected features ($\rho_s, \rho_{ad}, d_1$ and $d_2$) to conduct CorrTest. Otherwise, maximum likelihood estimates of branch lengths were obtained in MEGA 7 command line version (Kumar et al. 2012; Kumar et al. 2016) using the published topology, sequence alignments, and the substitution model specified in the original article. To examine the impact of the specification of a time-reversible substitution model on CorrTest, we estimated branch lengths under an unrestricted substitution model (Yang 1994) for all the nucleotide datasets in PAML (Yang 2007) and conducted CorrTest.

To obtain the autocorrelation parameter ($\nu$), we used MCMCTree (Yang 2007) with the same input priors as the original study, but omitting calibration priors to avoid the influence of calibration uncertainty densities on the estimate of $\nu$. We did, however, provide a root calibration because MCMCTree required it. For this purpose, we specified the root calibration as the one used in the original article or as the median age of the root node in the TimeTree database (Hedges et al. 2006; Kumar et al. 2017) ± 50My (uniform distribution with 2.5% relaxation on minimum and maximum bounds). Bayesian analyses required long computational times, so we used the original alignments in MCMCTree to infer $\nu$ if alignments were shorter than 20,000 sites. If the alignments were longer than 20,000 sites, we randomly selected 20,000 sites from the original alignments. However, one dataset (Ruhfel et al. 2014) contained more than 300 ingroup species, such that even alignments of 20,000 sites required prohibitive amounts of memory. In this case, we randomly selected 2,000 sites from the original alignments to use in MCMCTree for $\nu$ inference (similar results were obtained with a different site subset). Two independent runs of 5,000,000 generations each were conducted, and results were checked in Tracer (Rambaut et al. 2014) for convergence. ESS values were higher than 200 after removing 10% burn-in samples for each run. All empirical datasets are available at https://github.com/cathyqqtao/CorrTest.

Acknowledgements

We thank Xi Hang Cao for assisting on building the machine learning model, and Drs. Bui Quang Minh, Beatriz Mello, Heather Rowe, Ananias Escalante, Maria Pacheco, Jose Barba Montoya, Antonia Chroni, and S. Blair Hedges for critical comments and editorial
suggestions. This research was supported by grants from National Aeronautics and Space Administration (NASA NNX16AJ30G), National Institutes of Health (GM0126567-01; LM012487-03), National Science Foundation (NSF 1661218), and Tokyo Metropolitan University (DB105).
References


metabolism in Cyanobacteria. BMC Genomics 15:977.


Feng Y-J, Blackburn DC, Liang D, Hillis DM, Wake DB, Cannatella DC, Zhang P. 2017. Phylogenomics reveals rapid, simultaneous diversification of three major clades of


Res. 12:2825–2830.


Table 1. Patterns of rate autocorrelation inferred using the CorrTest approach.

<table>
<thead>
<tr>
<th>Taxonomic Group</th>
<th>Data type</th>
<th>Sequence count&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Sequence length</th>
<th>Substitution model</th>
<th>Rate model&lt;sup&gt;b&lt;/sup&gt;</th>
<th>CorrScore</th>
<th>P-value</th>
<th>1/ν&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mammals (A)</td>
<td>Nuclear</td>
<td>138</td>
<td>1,671</td>
<td>GTR + Γ</td>
<td>ABR &amp; IBR</td>
<td>0.98</td>
<td>&lt; 0.001</td>
<td>3.21</td>
<td>Meredith et al. (2011)</td>
</tr>
<tr>
<td>Mammals (B)</td>
<td>Nuclear</td>
<td>138</td>
<td>11,010</td>
<td>GTR + Γ</td>
<td>ABR &amp; IBR</td>
<td>0.99</td>
<td>&lt; 0.001</td>
<td>4.42&lt;sup&gt;d&lt;/sup&gt;</td>
<td>Meredith et al. (2011)</td>
</tr>
<tr>
<td>Mammals (C)</td>
<td>Nuclear</td>
<td>138</td>
<td>11,010</td>
<td>JTT + Γ</td>
<td>ABR &amp; IBR</td>
<td>0.99</td>
<td>&lt; 0.001</td>
<td>3.11</td>
<td>Meredith et al. (2011)</td>
</tr>
<tr>
<td>Mammals (D)</td>
<td>Mitochondrial DNA</td>
<td>271</td>
<td>7,370</td>
<td>HKY + Γ</td>
<td>ABR</td>
<td>0.98</td>
<td>&lt; 0.001</td>
<td>3.77&lt;sup&gt;e&lt;/sup&gt;</td>
<td>dos Reis et al. (2012)</td>
</tr>
<tr>
<td>Birds (A)</td>
<td>Nuclear DNA</td>
<td>198</td>
<td>101,781</td>
<td>GTR + Γ</td>
<td>IBR</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>2.07&lt;sup&gt;f&lt;/sup&gt;</td>
<td>Prum et al. (2015)</td>
</tr>
<tr>
<td>Birds (B)</td>
<td>Nuclear 3rd codon positions</td>
<td>222</td>
<td>1,364</td>
<td>GTR + Γ</td>
<td>IBR</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>2.11</td>
<td>Claramunt and Cracraft (2015)</td>
</tr>
<tr>
<td>Birds (C)</td>
<td>Nuclear 1&lt;sup&gt;st&lt;/sup&gt; and 2&lt;sup&gt;nd&lt;/sup&gt; codon positions</td>
<td>222</td>
<td>2,728</td>
<td>GTR + Γ</td>
<td>IBR</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>2.53</td>
<td>Claramunt and Cracraft (2015)</td>
</tr>
<tr>
<td>Insects</td>
<td>Nuclear proteins</td>
<td>143</td>
<td>220,091</td>
<td>LG + Γ</td>
<td>IBR</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>8.68&lt;sup&gt;g&lt;/sup&gt;</td>
<td>Misof et al. (2014)</td>
</tr>
<tr>
<td>Metazoans</td>
<td>Mitochondrial &amp; nuclear proteins</td>
<td>113</td>
<td>2,049</td>
<td>LG + Γ</td>
<td>ABR</td>
<td>0.65</td>
<td>&lt; 0.05</td>
<td>40.0</td>
<td>Erwin et al. (2011)</td>
</tr>
<tr>
<td>Plants (A)</td>
<td>Plastid 3rd codon positions</td>
<td>335</td>
<td>19,449</td>
<td>GTR + Γ</td>
<td>NA</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>2.28</td>
<td>Ruhfel et al. (2014)</td>
</tr>
<tr>
<td>Plants (B)</td>
<td>Plastid proteins</td>
<td>335</td>
<td>19,449</td>
<td>JTT + Γ</td>
<td>NA</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>2.46</td>
<td>Ruhfel et al. (2014)</td>
</tr>
<tr>
<td>Plants (C)</td>
<td>Nuclear 1&lt;sup&gt;st&lt;/sup&gt; and 2&lt;sup&gt;nd&lt;/sup&gt; codon positions</td>
<td>99</td>
<td>290,718</td>
<td>GTR + Γ</td>
<td>NA</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>5.50</td>
<td>Wickett et al. (2014)</td>
</tr>
<tr>
<td>Plants (D)</td>
<td>Chloroplast and nuclear DNA</td>
<td>124</td>
<td>5,992</td>
<td>GTR + Γ</td>
<td>IBR</td>
<td>1.00</td>
<td>&lt; 0.001</td>
<td>2.64</td>
<td>Beaulieu et al. (2015)</td>
</tr>
<tr>
<td>Fungi</td>
<td>Nuclear proteins</td>
<td>85</td>
<td>609,772</td>
<td>LG + Γ</td>
<td>NA</td>
<td>0.97</td>
<td>&lt; 0.001</td>
<td>3.78</td>
<td>Shen et al. (2016)</td>
</tr>
<tr>
<td>Parasitic protozoans</td>
<td>Mitochondrial DNA</td>
<td>91</td>
<td>6,863</td>
<td>HKY + Γ</td>
<td>ABR &amp; IBR</td>
<td>0.87</td>
<td>&lt; 0.01</td>
<td>2.41</td>
<td>Pacheco et al. (2018)</td>
</tr>
<tr>
<td>Prokaryotes (A)</td>
<td>Nuclear proteins</td>
<td>197</td>
<td>6,884</td>
<td>JTT + Γ</td>
<td>ABR</td>
<td>0.79</td>
<td>&lt; 0.05</td>
<td>2.54</td>
<td>Battistuzzi and Hedges (2009)</td>
</tr>
<tr>
<td>Prokaryotes (B)</td>
<td>Nuclear proteins</td>
<td>126</td>
<td>3,145</td>
<td>JTT + Γ</td>
<td>NA</td>
<td>0.83</td>
<td>&lt; 0.05</td>
<td>1.23</td>
<td>Calteau et al. (2014)</td>
</tr>
</tbody>
</table>

<sup>a</sup>Counts exclude outgroup taxa.  
<sup>b</sup>The branch rate model used in the original study. ABR: autocorrelated branch rate model; IBR: independent branch rate model; NA: no rate model information available.  
<sup>c</sup>1/ν is the inverse of the autocorrelation parameter that is estimated by MCMCTree using the ABR model in the time unit of 100My.  
<sup>d</sup>γ1/ν were
2.13 and 2.09 for each subtree in Mammals (B); 3.73, 1.04, and 2.47 for each subtree in Mammals (D); 1.60 and 2.07 for each subtree in Birds (A); 17.24 and 9.62 for each subtree in Insects.
Figure Legends

Figure 1. A flowchart showing an overview of the machine learning (McL) approach applied to develop the predictive model (CorrTest). We generated (a) 1,000 training datasets that were simulated using independent branch rate (IBR) models and (b) 1,000 training datasets that were simulated using autocorrelated branch rate (ABR) models. The numerical state (c) for all IBR datasets was 0 and (d) for all ABR datasets was 1. For each dataset, we estimated a molecular phylogeny with branch lengths (e and f) and computed $\rho_s$, $\rho_{ad}$, $d_1$, and $d_2$ (g and h) that served as features during the supervised McL. (i) Supervised McL was used to develop a predictive relationship between the input features and numerical states. (j) The predictive model produces a CorrScore for an input phylogeny with branch lengths. The predictive model was (k) validated with 10-fold and 2-fold cross-validation tests, (l) tested using external simulated data, and then (m) applied to empirical data to examine the prevalence of rate autocorrelation in the tree of life.

Figure 2. An evolutionary tree showing branch lengths (b), lineage lengths ($L$), lineage rates ($R$), and node times ($t$). $L_a = b_5 + \sqrt{b_1 b_2}$ and $L_b = b_6 + \sqrt{b_3 b_4}$. Relative lineage rates are computed from branch lengths using equations [28] - [31] and [34] - [39] in Tamura et al. (2018). Node times and branch rates are not required for estimating relative lineage rates.

Figure 3. The relationship of (a) ancestral and direct descendant lineage rates and (b) sister lineage rates when the simulated evolutionary rates were autocorrelated with each other (red) or varied independently (blue). The correlation coefficients are shown. (c) The decay of correlation between ancestral and descendant lineages when we skip one intervening branch ($d_1$) and when we skip two intervening branches ($d_2$). Percent decay values are shown. (d) Receiver Operator Characteristic (ROC) and Precision Recall (PR) curves (inset) of CorrTest for detecting branch rate model by using only the feature of ancestor-descendant lineage rates correlation ($\rho_{ad}$, green), only the feature of sister lineage rates correlation ($\rho_s$, orange), and all four features (all, black). The area under the curve is provided. (e) The relationship between the CorrScore produced by the machine.
learning model and the $P$-value. Independent branch rate model can be rejected when the CorrScore is greater than 0.83 at a significant level of $P < 0.01$, or when the CorrScore is greater than 0.5 at $P < 0.05$.

**Figure 4.** The performance of CorrTest in detecting autocorrelated and independent branch rate models in the analysis of datasets (Tamura et al. 2012) that were simulated with different (a) G+C contents, (b) transition/transversion ratios, and (c) average molecular evolutionary rates. The evolutionary rates are in the units of $10^{-3}$ substitutions per site per million years. (d–f) Patterns of CorrTest accuracy for data subsets containing 50, 100, 200, 300 and 400 ingroup sequences. The accuracy of CorrTest for different sequence lengths is shown when (d) the correct topology was assumed and (e) the topology was inferred. (f) The accuracy of CorrTest for datasets in which the inferred topology contained small and large number of topological errors. Darker color indicates higher accuracy.

**Figure 5.** Comparisons of the performance of CorrTest and Bayes factor analyses. (a) Distributions of 2 times the differences of marginal log-likelihood ($2\ln K$) estimated under independent branch rate (IBR) and autocorrelated branch rate (ABR) models via stepping-stone sampling method for datasets that were simulated under ABR (red) models and IBR (blue) models. ABR model is preferred ($P < 0.05$) when $2\ln K$ is greater than 3.841 (ABR zone), and IBR model is preferred when $2\ln K$ is less than -3.841 (IBR zone). When $2\ln K$ is between -3.841 and 3.841, the fit of the two rate models is not significantly different (gray shade). (b) The distributions of CorrScores in analyses of ABR (red) and IBR (blue) datasets. Rates are predicted to be autocorrelated if the CorrScore is greater than 0.5 ($P < 0.05$, ABR zone) and vary independently if the CorrScore is less than 0.5 (IBR zone). (c) The rate of detecting ABR model correctly (true positive rate) at different levels of statistical significance in Bayes factor (BF-SS) and CorrTest analyses. Posterior probabilities for ABR in BF-SS analysis are derived using the log-likelihood patterns in panel a. CorrTest $P$-values are derived using the CorrScore pattern in panel b. (d) The accuracy of identifying ABR model for datasets simulated with low ($\nu \geq 0.2$), moderate ($0.1 \leq \nu < 0.2$), and high ($\nu < 0.1$) levels of rate autocorrelation in Kishino et al.'s
The accuracy of identifying IBR model for datasets simulated at different degrees of rate variation in Drummond et al. (2006): low (standard deviation < 0.2), moderate (0.2 ≤ standard deviation < 0.3), and high (standard deviation ≥ 0.3).

Figure 6. The distribution of CorrScore for datasets (Tamura et al. 2012) with different taxon sampling densities. The CorrScore decreases when the density of taxon sampling is lower, as there is much less information to discriminate between ABR and IBR models. Red dashed lines mark two statistical significance levels of 5% and 1%. Results are summarized from 100 simulated datasets for each taxon sampling category.

Figure 7. The relationship between the inferred autocorrelation parameter (v) from MCMCTree and the true value for datasets simulated under autocorrelated branch rate models with the true v ranging from 0.01 to 0.3. The gray line represents the best-fit regression line, which has a slope of 1.09.

Figure 8. Patterns of CorrTest accuracy using the specific thresholds determined by predictive models trained with different ranges of the number of sequences (n): M100 (n ≤ 100), M200 (100 < n ≤ 200), M300 (200 < n ≤ 300), and M400 (n > 300) for the corresponding test datasets (Tamura et al. 2012). Accuracies are shown for 50, 100, 200, 300, and 400 ingroup sequences. The accuracy of CorrTest for different sequence lengths is shown when (a) the correct topology was assumed and (b) the topology was inferred. (c) The accuracy of CorrTest for datasets in which the inferred topology contained small and large number of topological errors. Darker color indicates higher accuracy.
Figure 3
Figure 4
Figure 5
Figure 6
Figure 7
Figure 8